# Precision QCD Predictions for Event-Shape Variables: Conjugate-Space Analysis and Determination of $\alpha_s$

## Scientific Context and Objectives of the Project

Event-shape variables in $e^+e^-$ annihilation, most notably the Thrust $(1-T)$ and the $C$-parameter, represent a uniquely clean arena for testing Quantum Chromodynamics (QCD) at the highest level of theoretical precision. Measured with exceptional statistical accuracy at LEP, they encode the full dynamics of multi-parton final states and are sensitive simultaneously to the perturbative running of the strong coupling and to non-perturbative hadronisation effects. Their theoretical description requires the consistent combination of three distinct ingredients: fixed-order perturbation theory, all-order resummation of large infrared logarithms, and a model for power-suppressed non-perturbative corrections.

One of the most important applications of this theoretical programme is the extraction of the strong coupling constant $\alpha_s(M_Z)$. Despite the apparent simplicity of the $e^+e^-$ environment, current extractions from event shapes reveal a persistent and troubling feature: different observables, analysed at the same perturbative accuracy, yield values of $\alpha_s(M_Z)$ that are, in some cases, mutually inconsistent at the level of about three standard deviations, and that tend to fall below the world average [1]. This tension is widely attributed to an incomplete treatment of non-perturbative corrections and to a methodological limitation that has received comparatively little attention: the standard practice of confronting theory with data exclusively in *physical space*, i.e. directly in the distribution of the event-shape variable itself.

The central scientific objective of this doctoral research project is to develop a new, theoretically rigorous approach to the analysis of event-shape distributions and to the extraction of $\alpha_s$, based on working directly in *conjugate (Laplace) space*, where kinematic factorisation is exact, and comparing with suitably transformed experimental data. This strategy, combined with state-of-the-art perturbative calculations and a systematic treatment of non-perturbative power corrections, is expected to yield a significant reduction of theoretical uncertainties and to provide a sharper, more transparent picture of the sources of inter-observable discrepancies.

The programme is also strongly motivated by the long-term perspective of future high-luminosity lepton colliders, FCC-ee [2] and CEPC [3], whose projected statistics will make the precision of theoretical predictions the limiting factor in any QCD analysis.

## Starting Point and Adopted Methodologies

**The case for conjugate-space analysis.** The Sudakov resummation of infrared logarithms in event-shape distributions is most naturally formulated in the Laplace conjugate variable $N$ (conjugate to the event-shape variable $\tau = 1-T$). In this space, the all-order resummed form factor takes a purely multiplicative form, soft and collinear contributions factorise exactly, and the resummed exponent can be computed systematically order by order in $\alpha_s \ln N$. Physical-space distributions are then obtained by an inverse Laplace transform, a step that introduces non-trivial complications: the integrand develops a Landau singularity that must be handled via a prescription (such as the Minimal Prescription of [4]), numerical inverse transforms near the kinematic boundaries of the distribution are delicate, and the exact kinematic endpoints, which are highly non-trivial functions of the parton multiplicity, as established by detailed recent studies [5, 6], can lead to boundary artefacts that contaminate fits.

Working instead directly in $N$-space, fitting the Laplace transform of experimental distributions against the theoretical prediction in conjugate space, eliminates these issues at their source. Kinematic factorisation is exact, power corrections enter as a multiplicative factor in $N$-space (a directly interpretable non-perturbative shape function), and the problematic boundary effects are avoided. This approach is well established in related contexts (transverse momentum resummation, deep inelastic scattering), but has not been systematically applied to event shapes, in part because transforming experimental data into $N$-space requires careful treatment of statistical and systematic uncertainties. Developing this methodology, including a rigorous uncertainty propagation framework for the Laplace-transformed data, constitutes a central technical contribution of this project.

**Perturbative ingredients.** The project exploits the state-of-the-art fixed-order and resummed calculations currently available for event shapes: NNLO distributions from parton-level generators such as NNLOJET [7], combined with N$^3$LL resummation within the CTTW framework [8], recently completed for Thrust [5]. A primary goal is to extend analogous N$^3$LL accuracy to the $C$-parameter and to additional global shapes, consolidating a coherent set of NNLO+N$^3$LL predictions formulated natively in $N$-space and available for systematic fitting.

**Non-perturbative power corrections.** Two complementary approaches to non-perturbative modelling will be implemented and compared within the conjugate-space framework. The first is the dispersive framework of Dokshitzer, Marchesini and Webber (DMW) [9], in which the leading $1/Q$ power correction is encoded in a universal infrared matrix element $\alpha_0(\mu_I)$; in $N$-space, this correction modifies the resummed exponent by a term linear in $N/Q$, which is particularly transparent. The universality of $\alpha_0$ across observables, up to calculable Milan factors [10], will be exploited in a global multi-observable fit. The second approach follows Nason and Zanderighi [11, 12], who compute non-perturbative corrections in the three-jet region using a renormalon analysis [13], avoiding the extrapolation of resummed predictions into regions where they are not reliable. The comparison of the two approaches within a common $N$-space perturbative framework will provide a robust, model-independent estimate of theoretical uncertainties.

**Hadron-mass scheme dependence and additional systematics.** Experimental distributions are sensitive to the scheme used to handle the finite masses of final-state hadrons, a source of uncertainty identified as potentially dominant in recent fits [12]. The three mass-handling schemes of Salam and Wicke will be incorporated in the analysis. The analytic coupling formulation [14, 15] will be explored as an alternative infrared treatment, replacing the perturbative coupling, which develops a Landau pole, with its unique infrared-safe analytic continuation.

**Multi-energy global fit.** Theoretical predictions will be matched to experimental data from $e^+e^-$ experiments spanning energies from 14 to 207 GeV, encompassing JADE, PETRA, SLC, and LEP. The energy evolution of power corrections provides a critical constraint on their $1/Q$ scaling and on the consistency of the non-perturbative models across the kinematic range.

## Expected Results

**Conjugate-space framework for event-shape analyses.** The primary scientific output will be a complete, self-consistent methodology for performing event-shape analyses directly in Laplace conjugate space. This includes theoretical predictions formulated in $N$-space at NNLO+N$^3$LL accuracy, a framework for transforming experimental distributions with full uncertainty propagation, and validated fitting procedures for $\alpha_s$ and the non-perturbative parameters.

**Precision extraction of $\alpha_s$.** The global fit, free from physical-space boundary effects and with systematically assessed non-perturbative uncertainties, is expected to yield a determination of $\alpha_s(M_Z)$ with a total uncertainty at the sub-percent level. The cross-comparison of the DMW and Nason–Zanderighi models within the same framework will quantify the degree to which current inter-observable tensions are of perturbative versus non-perturbative origin.

**Predictions for FCC-ee and CEPC.** The calibrated framework will be used to generate state-of-the-art event-shape predictions at the energies foreseen for FCC-ee (91, 160, 240, 365

GeV) and CEPC, providing essential theoretical benchmarks for the precision QCD programmes at these facilities.

### Logistical Context and Co-Supervisions

The successful execution of this project requires a broad set of expertises that spans from analytic resummation to numerical simulation of particle interactions and handling of experimental data. Therefore, it will be developed in the context of an international collaboration which includes Matteo Cacciari (Université Paris Cité, LPTHE) and Giancarlo Ferrera (Università di Milano, Italy). The supervisors' expertise, encompassing analytic resummation in QCD, fixed order calculations, power corrections, jet physics, numerical analyses, and more, overlaps and complements each other effectively. On the one hand, this ensures that the proposed research lays on solid foundations and that relevant skills are readily available in the team. On the other hand, the presence of two supervisors will enrich the student experience, allowing them to be part of an international team from the very beginning of their PhD programme. We foresee multiple research visits during the course of this PhD and we expect to set up a formal 'co-tutelle' between Università di Milano and Université Paris Cité, with the student expected to spend at least one year at each institution.

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